

# Franck-Condon Factors to High Vibrational Quantum Numbers I: $N_2$ and $N_2^+$

R. W. Nicholls<sup>1</sup>

(June 5, 1961)

Franck-Condon factor arrays have been computed numerically to high vibrational quantum numbers for the band systems

$N_2$ :	$C^3\Pi - B^3\Pi$	(Second Positive)
$N_2$ :	$B^3\Pi - A^3\Sigma$	(First Positive)
$N_2$ :	$A^3\Sigma - X^1\Sigma$	(Vegard Kaplan)
$N_2$ :	$a^1\Pi - X^1\Sigma$	(Lyman-Birge-Hopfield)
$N_2^+$ :	$A^2\Pi - X^2\Sigma$	(Meinel)
$N_2^+$ :	$B^2\Sigma - X^2\Sigma$	(First Negative)

and for the following ionization transitions

$$\begin{aligned} N_2 \quad X^1\Sigma &\rightarrow N_2^+ X^2\Sigma \\ N_2 \quad X^1\Sigma &\rightarrow N_2^+ A^2\Pi \\ N_2 \quad X^1\Sigma &\rightarrow N_2^+ B^2\Sigma \end{aligned}$$

## 1. Introduction

The intensity  $I_{v'v''}$  in emission of the  $(v', v'')$  molecular band is given by Nicholls (1958)

$$I_{v'v''} = KN_{v'} E_{v'v''}^4 R_e^2(\bar{r}_{v'v''}) q_{v'v''} \quad (1)$$

where  $K$  is a constant which contains the effects of geometry and units,  $N_{v'}$  is the population in the level  $v'$ ,  $E_{v'v''}$  is the energy quantum of the  $v', v''$  transition,  $R_e(r)$  is the electronic transition moment.  $r$  is the internuclear separation and  $\bar{r}_{v'v''}$  is the  $r$ -centroid (Nicholls and Jarman, 1956, 1959) of the transition and  $q_{v'v''}$  is the Franck-Condon factor (Bates, 1952; Nicholls, 1958).  $R_e(r)$ ,  $\bar{r}_{v'v''}$  and  $q_{v'v''}$  are respectively defined by:

$$R_e(r) = \int \psi_e'^* M \psi_e'' d\tau_e \quad (2)$$

$$\bar{r}_{v'v''} = \int \psi_{v'} \psi_{v''} r dr / \int \psi_{v'} \psi_{v''} dr \quad (3)$$

$$q_{v'v''} = \left| \int \psi_{v'} \psi_{v''} dr \right|^2 \quad (4)$$

where  $\psi_e'$ ,  $\psi_e''$  are respectively electronic wave functions of the upper and lower states involved,  $M$  is the multipole moment of the transition,  $d\tau_e$  is the element of configuration space of electronic wave function,  $r$  is the internuclear separation, and  $\psi_{v'}$ ,  $\psi_{v''}$  are the vibrational wave functions of the upper ( $v'$ ) level and lower ( $v''$ ) level, respectively.

The Franck-Condon factor ( $q_{v'v''}$ ) is governed by the overlap of the vibrational wave functions of the two levels which are concerned in the  $v', v''$  transition. It is sometimes called, somewhat clumsily, the vibrational overlap integral square and can vary over two or three orders of magnitude for the significant bands of a system. For many band systems, although the variation from band to band of  $N_{v'}$ ,  $E_{v'v''}$  and  $R_e(r)$  is significant, their combined effect is often not more than one or two orders of magnitude.  $E$  does not change very much over a band system of relatively small wavelength extension (e.g.,  $\Delta E^4/E^4 = 14$  percent between 2000 and 6000 Å and few band systems are so extensive).  $N_{v'}$  is of course controlled by excitation conditions. For a number of band systems which have been studied, a significant variation  $R_e(r)$  with  $r$  has been measured, (Turner and Nicholls, 1954a, b; Wallace and Nicholls, 1955; Nicholls 1956; Dixon and Nicholls, 1958; Robinson and Nicholls, 1958, 1960; Hebert and Nicholls, 1961). It is however seldom as much as an order of magnitude. Thus the Franck-Condon factor exerts dominant effect upon the intensity variation from band to band across a system.

Since the pioneering work of Hutchisson (1930, 1931) who used an analytic method to evaluate the vibrational overlap integral for homonuclear molecules (harmonic and anharmonic oscillators), and its extension by Dunham (1932) to heteronuclear molecules, a number of authors have used a wide variety of methods and have produced a number of  $q_{v'v''}$  arrays of varying extent. This work may be summarized as follows:

Gaydon and Pearce (1939) proposed a method by which wave functions of a simple harmonic oscillator are linearly distorted to fit an equivalent Morse

<sup>1</sup> Department of Physics, University of Western Ontario, London, Ontario. Consultant in Heat Division, National Bureau of Standards, 1959–60.

Potential. They compared Franck-Condon factors calculated in this way with their experimental results on RbH. Pillow (1949, 1950, 1951, 1952, 1953a,b, 1954, 1955) has improved and applied the method to a large number of transitions (Pillow and Rowlett, 1960). Nicholls (1950), Montgomery and Nicholls (1951) and Turner and Nicholls (1951) also used the method.

The basis of the distortion method was criticized by Wu (1952) who suggested two further analytic methods for Morse molecules. One of these methods which is based on the WKB approximation has been used by Wyller (1953, 1958). Bates (1952) has published a very useful double entry table based upon modifications of Hutchisson's formulas, from which Franck-Condon factors for any transition may be read  $v'=0-2$ ;  $v''=0-2$ .

Prior to the general availability of digital computers, the extreme tedium of direct numerical integration, such as has been carried out by Bates (1949) and Jarmain and Nicholls (1954) for Morse molecules, forced most workers to resort to analytic approximations to overlap integrals. Most of these approximations are valid for small quantum numbers. An example is the analytic method of Fraser and Jarmain (Fraser and Jarmain, 1953; Jarmain and Fraser, 1953; Fraser, 1954) which has been applied to a wide variety of band systems (Jarmain, Fraser, and Nicholls, 1953, 1955; Fraser, Jarmain, and Nicholls, 1954; Nicholls, Fraser, and Jarmain, 1959; Nicholls, Fraser, and Jarmain, and McEachran, 1960; Jarmain, Ebisuzaki, and Nicholls, 1960; Ortenberg, 1960).

Biberman and Yakubov (1960) have recently proposed a WKB method of evaluating Franck-Condon factors analytically at high vibrational quantum numbers. The method has been applied by Yakubov (1960). Nevertheless, with the current availability of large capacity electronic computers, direct numerical integration to high quantum numbers is straightforward, and there seems to be little point in developing further approximate analytic methods. The only limitation of this method is the realism (particularly at high quantum numbers) of the empirical potentials used.

Losev (1958) used a small capacity computer for direct evaluation of overlap integrals. The capacity of the machine was not, however, sufficient to accommodate the severe cancellation.

Ideally, Franck-Condon factor arrays computed to as high vibrational quantum numbers  $v'$  and  $v''$  as are physically reasonable from observed spectra are required for all important molecular band systems. This implies good knowledge of molecular potentials so that the wave functions used are appropriate to 'real' rather than empirical potentials. Most of the Franck-Condon factor tables currently in the literature have been calculated for empirical (e.g., simple harmonic or Morse oscillator) potentials.

However recent work of Jarmain (1959, 1960) and Vanderslice et al. (Vanderslice, Mason and Maisch, 1959, 1960a, 1960b; Tobias, Fallon, and Vanderslice, 1960; Fallon, Tobias, and Vanderslice, 1961) has provided, by a WKB-Klein-Dunham method, numerical

information on 'real' molecular potentials. It is therefore planned to program a numerical solution of the Schrodinger equation appropriate to such numerical potentials to provide numerical wave functions from which more realistic Franck-Condon factor arrays can be determined.

## 2. Method

In the meantime the results reported in this paper represent an interim step. Morse potentials have been assumed and a program for the numerical integration of the overlap integrals has been written for the electronic digital 704 computer at the National Bureau of Standards. The input data are  $\omega_e$ ,  $\omega_e x_e$ ,  $r_e$ ,  $\mu_A$ ,  $v_{max}$  for both of the electronic states involved. Each member of the two families of wave functions  $\psi'_i$  ( $i=0-v'_{max}$ ),  $\psi''_j$  ( $j=0-v''_{max}$ ) is computed at 0.01A intervals, overlap integrals between all pairs of them are computed and squared. The program has been applied to a number of transitions and results are presented here for transitions in  $N_2$  and  $N_2^+$ .

The basis of the method is as follows: The zeroth vibrational wave function of a Morse molecule is (Morse 1929).

$$\psi_0(r) = \left( \frac{\alpha}{\Gamma(K-1)} \right)^{1/2} [K \exp(-\alpha \overline{r-r_e})]^{1/2(K-1)} \exp \left[ -\frac{K}{2} \exp(-\alpha \overline{r-r_e}) \right] \quad (5)$$

$$\text{where } K = \frac{\omega_e}{\omega_e x_e}$$

$$\alpha = 0.243534 (\mu_A \omega_e x_e)^{1/2} A^{-1}$$

$r_e$  = equilibrium internuclear separation (A)

Thus the input data for point by point computation of  $\psi_0(r)$  at a series of values of  $r$  is  $\omega_e$ ,  $\omega_e x_e$ ,  $r_e$ ,  $\mu_A$ . The symbolism is standard (Herzberg, 1950). The gamma function may be evaluated through Stirling's formula:

$$\Gamma(x) = \left( \frac{2\pi}{x} \right)^{1/2} x^x \exp(-x) \left\{ 1 + \frac{1}{12x} + \frac{1}{288x^2} + \dots \right\} \quad (6)$$

Once the zeroth wave function is tabulated the complete family of wave functions can be built up one at a time by the relationships

$$\left( \frac{\psi_v}{\psi_{v-1}} \right) = \left( \frac{C_v}{C_{v-1}} \right) \frac{1}{Z} \left( \frac{L_v}{L_{v-1}} \right) \quad (7)$$

$$= \left[ \frac{(K-v)(K-2v-1)}{v(K-2v+1)} \right]^{1/2} \times \frac{1}{K \exp(-\alpha \overline{r-r_e})} \left( \frac{L_v}{L_{v-1}} \right) \quad (8)$$

$$C_0 = \left( \frac{\alpha}{\Gamma(K-1)} \right)^{1/2} \quad (9)$$

The wave functions  $\psi_v$  of higher quantum number than zero contain, as is seen in eq (7), ratios of associated Laguerre functions  $L_v$  of the form:

$$L_v = L_{K-v-1}^{K-2v-1}(Z) = Z^v - v(K-v-1)Z^{v-1} + \frac{v(v-1)}{2}(K-v-1)(K-v-2)Z^{v-2} + \dots + (-1)^v \{(K-v-1)(K-v-2) \dots (K-2v)\} \quad (10)$$

where the number of terms in the  $v$ th polynomial is  $(v+1)$ ;  $L_0 = L_{K-1}^{K-1}(Z) = 1$ ;  $Z = K \exp(-\alpha r - r_e)$ . For the lowest quantum numbers, eq (10) may be used in the evaluation of  $L_v$ . At higher values of  $v$  loss of significant figures through cancellation between positive and negative terms forces the use of the recursion relation

$$L_v(Z) = ZL_{v-1} - (K-2v) \sum_{r=0}^{v-1} \binom{v-1}{r} \frac{(2r)!}{(r+1)!} L_{v-r-1} \quad (11)$$

(Fraser, Jarman, and Henderson, 1959). If in the use of eq (8)  $L_{v-1}$  should vanish at some value of  $r$ , the alternative relation

$$\left(\frac{\psi_v}{\psi_{v-2}}\right) = \left(\frac{C_v}{C_{v-2}}\right) \left(\frac{1}{Z^2}\right) \left(\frac{L_v}{L_{v-2}}\right) \quad (12)$$

can be used.

Prior to numerical integration, each wave function is evaluated at 0.01A intervals over its significant range of  $r$ . This significant range may be estimated by recalling that the width  $r_2 - r_1$  of a Morse potential in the region of energy  $U$  is given by

$$r_2 - r_1 = \frac{1}{\alpha} \log_e \left[ \frac{1 + \left(\frac{U}{D}\right)^{1/2}}{1 - \left(\frac{U}{D}\right)^{1/2}} \right] = \frac{1}{\alpha} \log_e \left[ \frac{1 + 2(V - V^2)^{1/2}}{1 - 2(V - V^2)^{1/2}} \right] \text{ at level } v \text{ where } V = x_e(v + 1/2). \quad (13)$$

Rough rules of thumb for the total range in  $r$  to be treated are for many electronic states

$v_{\max} \sim 6$  Add 50% to  $r_2 - r_1$ : 40% of total range should be less than  $r_e$   
 $v_{\max} \sim 12$  Add 40% to  $r_2 - r_1$ : 40% of range should be less than  $r_e$   
 $v_{\max} \sim 18$  Add 30% to  $r_2 - r_1$ : 35% of range should be less than  $r_e$ .

Beyond these ranges the amplitudes of wave functions are negligible.

Provision has been made for storage on magnetic tape of the vibrational wave functions involved in the NBS computer center, for future users. Checks were of course applied for normality of the wave functions i.e.,  $\int (\psi_v)^2 dr = 1$  and the overlap integrals between all possible pairs of wave functions of a transition were evaluated. The overlap integrals were squared and values checked by applying the sum rules  $\sum_v q_{v'v''} = 1 = \sum_{v''} q_{v'v''}$ . It may be noted in passing that computation of  $r$ -centroids was also made. These will be discussed independently elsewhere.

### 3. Basic Data and Results

Franck-Condon factor arrays have been computed for the following radiative transitions

$$N_2: \begin{array}{ll} C^3\Pi_u - B^3\Pi_g & \text{Second Positive} \\ B^3\Pi_g - A^3\Sigma_u^+ & \text{First Positive} \\ A^3\Sigma_u^+ - X^1\Sigma_g^+ & \text{Vegard-Kaplan} \\ a^1\Pi_g - X^1\Sigma_g^+ & \text{Lyman-Birge-Hopfield} \end{array}$$

$$N_2^+: \begin{array}{ll} A^2\Pi_u - X^2\Sigma_g^+ & \text{Meinel} \\ B^2\Sigma_u^+ - X^2\Sigma_g^+ & \text{First Negative} \end{array}$$

and also for the following ionization transitions

$$N_2(X^1\Sigma_g^+, v=0) \rightarrow N_2^+ X^2\Sigma(v=0-21)$$

$$N_2(X^1\Sigma_g^+, v=0) \rightarrow N_2^+ A^2\Pi(v=0-5)$$

$$N_2(X^1\Sigma_g^+, v=0) \rightarrow N_2^+ B^2\Sigma(v=0-29)$$

Wave functions were computed in all cases on the basis of a Morse model for as many levels as were known spectroscopically and a rectangular  $q_{v'v''}$  array of  $(v'_{\max} + 1)(v''_{\max} + 1)$  members were then evaluated for each transition. In such arrays data was of course calculated for many bands which were not usually spectroscopically observed.

The input data ( $\omega_e$ ,  $\omega_e x_e$ ,  $r_e$ ,  $\mu_A$ ,  $v_{\max}$ ) used for each of the states is listed in table 1 and was taken in large part from the compilation of Mulliken (1959) and the original papers of analysis of the band systems.

The nine arrays of Franck-Condon factors are presented in tables 2 to 10 inclusive.

TABLE 1. Basic Data

State	$\omega_e(\text{cm}^{-1}) \times 10^{-3}$	$\omega_e x_e(\text{cm}^{-1}) \times 10^{-1}$	$r_e(\text{\AA})$	$\mu_A$	$v_{\max}$
$N_2 X^1\Sigma$	2.35807	1.419	1.0976	7.00377	27
$a^1\Pi$	1.6937	1.383	1.220	7.00377	16
$A^3\Sigma$	1.46037	1.3891	1.293	7.00377	16
$B^3\Pi$	1.73411	1.447	1.2123	7.00377	21
$C^3\Pi$	2.0351	1.708	1.1482	7.00377	4
$N_2^+ X^2\Sigma$	2.20719	1.614	1.118	7.00363	21
$A^2\Pi$	1.90284	1.491	1.177	7.00363	5
$B^2\Sigma$	2.41984	2.319	1.075	7.00363	29

TABLE 2. Franck-Condon factors to high vibrational quantum numbers for the  $N_2$  second positive ( $C^3\Pi - B^3\Pi$ ) band system

$v' \backslash v''$	0	1	2	3	4	5	6	7	8	9	10	11
0	4.4929-1	3.2870-1	1.4691-1	5.2258-2	1.6345-2	4.7260-3	1.2999-3	3.4632-4	9.0427-5	2.3327-5	5.9777-6	1.5275-6
1	3.8986-1	1.8685-2	2.0376-1	2.0027-1	1.1238-1	4.8391-2	1.7904-2	6.0242-3	1.9045-3	5.7747-4	1.7024-4	4.9252-5
2	1.3494-1	3.2232-1	3.2988-2	5.9570-2	1.6138-1	1.4274-1	8.3030-2	3.8599-2	1.5642-2	5.7932-3	2.0181-3	6.7377-4
3	2.3630-2	2.5146-1	1.6304-1	1.1812-1	1.8410-3	8.8906-2	1.3450-1	1.0624-1	6.1610-2	2.9799-2	1.2816-2	5.0889-3
4	2.1905-3	6.9566-2	3.0342-1	4.7523-2	1.5698-1	1.4160-2	2.9388-2	9.9550-2	1.1050-1	8.0226-2	4.6144-2	2.2924-2

  

$v' \backslash v''$	12	13	14	15	16	17	18	19	20	21
0	3.9014-7	9.9745-8	2.5539-8	6.5466-9	1.6780-9	4.2921-10	1.0925-10	2.7561-11	6.8697-12	1.6833-12
1	1.4075-5	3.9912-6	1.1265-6	3.1711-7	8.9119-8	2.5012-8	7.0058-9	1.9548-9	5.4183-10	1.4846-10
2	2.1835-4	6.9311-5	2.1688-5	6.7206-6	2.0690-6	6.3413-7	1.9374-7	5.9029-8	1.7929-8	5.4207-9
3	1.9106-3	6.8923-4	2.4164-4	8.3002-5	2.8101-6	9.4178-6	3.1341-6	1.0378-6	3.4236-7	1.1258-7
4	1.0317-2	4.3321-3	1.7309-3	6.6710-4	2.5045-4	9.2257-5	3.3524-5	1.2065-5	4.3127-6	1.5344-6

#### 4. Discussion

The most obvious feature of the double entry tables of Franck-Condon factors is, as indicated on them, the family of well defined loci on the  $v' v''$  plane of maximum values of the factors. The loci, which have the general appearance of co-axial parabolas correlate well with the occurrence of observed bands. These primary and subsidiary 'Condon Parabolas', as the loci are called, pass through those  $v', v''$  for which the largest contribution to each overlap integral arises from the strong overlap in  $r$  between one antinode of each of the wave functions  $\psi_{v'}$  and  $\psi_{v''}$ .

The wave function  $\psi_v(r)$  has  $v+1$  antinodes of which the general one is  $r_i^{(v)}$ .  $r_1^{(v)}$  and  $r_{v+1}^{(v)}$  are the terminal antinodes and  $r_1^{(v)} < r_e < r_{v+1}^{(v)}$ . The primary Condon parabolas arise, as is well known, from overlap of the terminal antinodes of  $\psi_{v'}$  and  $\psi_{v''}$ . The two branches of the parabola are determined by the conditions

$$\left. \begin{aligned} r_{v'+1}^{(v')} &= r_{v''+1}^{(v'')} \\ r_1^{(v')} &= r_1^{(v'')} \end{aligned} \right\} \quad (14)$$

The secondary parabolas are determined by overlap of the terminal antinode of one wave function and the next-to-terminal antinode of the other, that is:

$$\left. \begin{aligned} r_{v'}^{(v')} &= r_{v''+1}^{(v'')} \\ r_1^{(v')} &= r_2^{(v'')} \end{aligned} \right\} \quad (15)$$

The conditions for the  $i$ th subsidiary parabola are then:

$$\left. \begin{aligned} r_{v'+2-i}^{(v')} &= r_{v''+1}^{(v'')} \\ r_1^{(v')} &= r_i^{(v'')} \end{aligned} \right\} \quad (16)$$

The geometry of the parabolas has been studied for a variety of potentials by applying the conditions

of eqs (14), (15), and (16) and is discussed in detail elsewhere (Nicholls, 1961). In this work the dominating influence of  $\Delta r_e$  upon the shape of the 'Parabolas' is shown analytically. The loci are only true parabolas for simple harmonic potentials as Condon (1926) pointed out for the primary parabola. For Morse Potentials the primary 'parabola' is actually a quartic.

The following qualitative statements may be made upon the dominant influence of  $\Delta r_e$  upon the shape of the loci in particular and upon the form of the three dimensional Franck-Condon factor surface in general.

For band systems in which  $\Delta r_e$  is quite small, that is, the potentials lie essentially one above the other, the primary parabola is very narrow, appears to lie along the prime diagonal  $v'=v''$  in the  $v', v''$  plane and has its maximum value at the (0,0) band. In a three dimensional representation when  $\Delta r_e$  is small ( $\sim 0.01\text{\AA}$ ) the  $q_{v'v''}$  surface is often a descending diagonal ridge along  $v'=v''$ .

When  $\Delta r_e$  is a little larger ( $\sim 0.05\text{\AA}$ ) the ridge widens into a very narrow parabolic ridge with an axial valley. This situation occurs for the  $N_2^+$  First Negative band system.

As  $\Delta r_e$  is further increased, the primary parabola widens, its vertex moves down the prime diagonal away from (0,0), and subsidiary parabolas appear. At relatively large ( $\sim 0.4\text{\AA}$ ) values of  $\Delta r_e$  (e.g.,  $O_2$  Schumann-Runge and  $N_2$  Vegard-Kaplan systems) the band system is quite extended in wavelength and the oscillator strength of the whole system is spread over many more bands than for systems having a smaller  $\Delta r_e$ . Thus the intensity per band is decreased.

In an extreme case, the primary parabola lies along the  $v'=0$  and  $v''=0$  progressions at large  $v''$  and  $v'$  respectively and avoids the (0,0) region entirely.

Examples of these general remarks are seen in the tables. The  $N_2^+$  First Negative system having a small  $\Delta r_e$  (0.043\AA) exhibits a very narrow primary



TABLE 3. *Franck-Condon factors to high vibrational quantum numbers for the N<sub>2</sub> first positive (B<sup>3</sup>Π—A<sup>3</sup>Σ) band system*

$\begin{smallmatrix} v'' \\ v' \end{smallmatrix}$	0	1	2	3	4	5	6	7	8
0	3.3816-1	3.2480-1	1.8996-1	8.8567-2	3.6486-2	1.3990-2	5.1465-3	1.8513-3	6.5953-4
1	4.0645-1	2.3100-3	1.0321-1	1.7820-1	1.4502-1	8.6473-2	4.3670-2	1.9997-2	8.6259-3
2	1.9746-1	2.1203-1	1.1320-1	1.2048-3	7.7240-2	1.2750-1	1.1271-1	7.4955-2	4.2471-2
3	5.0143-2	2.9871-1	3.8683-2	1.6230-1	3.2267-2	9.0504-3	6.9100-2	1.0094-1	9.0789-2
4	7.1905-3	1.3180-1	2.7381-1	1.8065-3	1.1388-1	8.8227-2	5.2273-3	1.7977-2	6.3607-2
5	5.8713-4	2.7286-2	2.1065-1	1.8081-1	4.7796-2	4.2616-2	1.0567-1	3.8289-2	1.2298-6
6	2.6156-5	2.9248-3	6.1481-2	2.6054-1	8.3053-2	1.0404-1	3.1711-3	8.0781-2	6.9671-2
7	5.6982-7	1.6132-4	8.4620-3	1.0650-1	2.7061-1	1.9163-2	1.2905-1	6.7482-3	3.9461-2
8	4.7221-9	4.1737-6	5.6752-4	1.8566-2	1.5609-1	2.4380-1	2.7659-5	1.1586-1	3.6330-2
9	6.5075-12	3.9626-8	1.7188-5	1.4947-3	3.4197-2	2.0292-1	1.9185-1	1.6939-2	7.8622-2
10	2.7802-15	6.0552-11	1.8482-7	5.2408-5	3.2741-3	5.5689-2	2.4021-1	1.2991-1	5.2188-2
11	8.1140-16	3.3224-14	3.0349-10	6.3216-7	1.3176-4	6.2985-3	8.2650-2	2.6301-1	7.2055-2
12	2.5148-16	5.0349-16	2.1376-13	1.1012-9	1.7690-6	2.8895-4	1.0992-2	1.1398-1	2.6378-1
13	5.9625-17	1.6620-16	4.3337-16	1.0512-12	3.2357-9	4.2896-6	5.7172-4	1.7773-2	1.4799-1
14	3.4245-16	4.4756-17	4.5421-16	9.7232-16	4.0476-12	8.1628-9	9.3348-6	1.0436-3	2.7016-2
15	1.7005-16	2.5344-16	2.7160-17	4.1387-16	2.8052-15	1.2846-11	1.8385-8	1.8657-5	1.7849-3
16	5.1720-18	7.4160-17	2.2234-16	4.9038-17	6.3948-17	4.5515-14	3.4427-11	3.7813-8	3.4803-5
17	2.0782-16	1.5676-17	2.1122-16	5.4009-16	2.1696-15	2.4366-15	2.1664-13	8.5321-11	7.2030-8
18	1.8889-16	1.2423-16	2.4354-17	9.1758-17	1.2590-16	3.5970-16	3.0869-17	4.8996-13	1.9965-10
19	1.0678-17	1.1379-16	1.9019-17	2.6078-16	9.9179-16	3.8594-15	4.2273-15	8.3215-15	1.2193-12
20	7.5363-17	2.8873-18	3.7751-16	2.6077-16	9.2651-17	9.5416-17	9.2691-16	1.7885-15	1.2898-14
21	1.7325-16	2.6683-17	4.2656-16	4.0688-18	4.6754-16	3.5655-15	3.9524-15	2.1349-15	9.2663-17

  

$\begin{smallmatrix} v'' \\ v' \end{smallmatrix}$	9	10	11	12	13	14	15	16
0	2.3477-4	8.4031-5	3.0383-5	1.1135-5	4.1473-6	1.5729-6	6.0831-7	2.4019-7
1	3.5891-3	1.4631-3	5.9070-4	2.3802-4	9.6271-5	3.9251-5	1.6185-5	6.7658-6
2	2.1815-2	1.0529-2	4.8845-3	2.2119-3	9.8835-4	4.3920-4	1.9525-4	8.7217-5
3	6.4061-2	3.9171-2	2.1871-2	1.1508-2	5.8243-3	2.8759-3	1.3995-3	6.7625-4
4	8.3488-2	7.5106-2	5.5103-2	3.5667-2	2.1275-2	1.2016-2	6.5431-3	3.4791-3
5	2.4543-2	5.8198-2	7.0564-2	6.3384-2	4.7949-2	3.2525-2	2.0514-2	1.2313-2
6	1.2893-2	2.4600-3	2.8099-2	5.2611-2	6.0394-2	5.4340-2	4.2250-2	2.9867-2
7	7.7327-2	3.8328-2	2.7384-3	6.4003-3	2.9121-2	4.7031-2	5.2119-2	4.7183-2
8	8.3601-3	6.0770-2	5.7211-2	1.8520-2	8.6772-5	9.5049-3	2.8339-2	4.1643-2
9	6.7727-2	3.1940-4	3.3048-2	5.9448-2	3.7289-2	7.9625-3	3.6806-4	1.1202-2
10	3.7645-2	8.3700-2	1.3068-2	9.5758-3	4.6033-2	4.8138-2	2.2815-2	3.0276-3
11	8.7792-2	9.0118-3	7.8966-2	3.5274-2	6.6872-5	2.5442-2	4.6325-2	3.5766-2
12	2.8566-2	1.1086-1	2.6171-5	5.8457-2	5.4227-2	5.7104-3	7.8299-3	3.3974-2
13	2.5751-1	4.6872-3	1.1561-1	9.3424-3	3.2354-2	6.1513-2	2.0820-2	1.3633-4
14	1.8258-1	2.3145-1	7.7329-4	1.0292-1	2.9820-2	1.0861-2	5.5268-2	3.6818-2
15	3.9015-2	2.1544-1	1.9452-1	1.3301-2	7.8290-2	5.2260-2	5.5460-4	3.9360-2
16	2.8927-3	5.3947-2	2.4432-1	1.5158-1	3.6337-2	4.9346-2	6.8675-2	2.9424-3
17	6.1314-5	4.4801-3	7.1852-2	2.6718-1	1.0776-1	6.3088-2	2.3488-2	7.4348-2
18	1.2872-7	1.0294-4	6.6745-3	9.2613-2	2.8243-1	6.7723-2	8.7269-2	6.1701-3
19	4.3255-10	2.1785-7	1.6584-4	9.6153-3	1.1595-1	2.8900-1	3.5244-2	1.0409-1
20	3.1035-12	8.8458-10	3.5155-7	2.5781-4	1.3450-2	1.4143-1	2.8644-1	1.2813-2
21	3.1514-14	7.4958-12	1.7175-9	5.4388-7	3.8848-4	1.8331-2	1.6845-1	2.7497-1

TABLE 4. Franck-Condon factors to high vibrational quantum numbers for the  $N_2$  Vegard-Kaplan ( $\Lambda^3\Sigma_g^+ - X^1\Sigma_g^+$ ) band system

$v' \backslash v''$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	5.9004-4	5.3372-3	2.2858-2	6.1668-2	1.1764-1	1.6884-1	1.8941-1	1.7036-1	1.2501-1	7.5777-2	3.8279-2	1.6212-2	5.7795-3	1.7379-3	4.4108-4
1	3.3186-3	2.2781-2	6.9190-2	1.1931-1	1.2164-1	6.3828-2	6.2561-3	1.3953-2	7.8661-2	1.3698-1	1.4493-1	1.0997-1	6.4036-2	2.9626-2	1.1103-2
2	9.9751-3	5.0849-2	1.0348-1	9.7316-2	2.9217-2	2.1608-3	5.4725-2	8.9237-2	4.6160-2	8.8402-4	2.9551-2	1.0011-1	1.3776-1	1.1956-1	7.4723-2
3	2.1326-2	7.8496-2	9.7675-2	3.3262-2	2.5390-3	5.6172-2	6.3910-2	8.5635-3	1.6968-2	7.3178-2	6.1805-2	7.6001-3	1.5200-2	8.3825-2	1.3146-1
4	3.6417-2	9.3276-2	5.9979-2	3.7143-4	4.1074-2	5.7583-2	4.6254-3	2.5535-2	6.3308-2	1.9160-2	7.1270-3	6.2759-2	6.3144-2	9.3831-3	1.3789-2
5	5.2901-2	8.9624-2	2.0109-2	1.4649-2	5.8682-2	1.2234-2	1.6881-2	5.4617-2	1.0645-2	1.6426-2	5.8712-2	2.0156-2	6.5852-3	6.1432-2	5.8318-2
6	6.7985-2	7.0820-2	8.3805-4	4.2328-2	3.4003-2	2.6303-3	4.7678-2	1.6412-2	1.1626-2	5.0081-2	9.7430-3	1.7291-2	5.5788-2	1.4565-2	1.1314-2
7	7.9391-2	4.5385-2	5.2638-3	5.1543-2	5.4018-3	2.8049-2	3.2372-2	1.7097-3	4.3075-2	1.3077-2	1.4090-2	4.6348-2	4.7851-3	2.4469-2	5.1703-2
8	8.5879-2	2.2052-2	2.2306-2	3.8028-2	1.9331-3	4.2032-2	4.1846-3	2.7007-2	2.5721-2	4.0727-3	4.1475-2	6.2965-3	2.1849-2	4.0018-2	3.6001-4
9	8.7302-2	6.4345-3	3.8377-2	1.6510-2	1.8150-2	2.8901-2	3.7627-3	3.6963-2	8.8547-4	3.1093-2	1.5917-2	1.0832-2	3.7647-2	6.3259-4	3.2836-2
10	8.4340-2	2.0527-4	4.5272-2	2.2543-3	3.3488-2	8.0539-3	2.2423-2	1.8373-2	9.7438-3	3.0255-2	5.2910-4	3.4748-2	5.3800-3	2.2182-2	2.7567-2
11	7.8120-2	2.0154-3	4.1796-2	9.8901-4	3.5295-2	1.6839-5	3.2569-2	1.2430-3	2.8347-2	7.1660-3	1.9634-2	1.8916-2	7.2235-3	3.1974-2	5.8293-6
12	6.9875-2	9.0148-3	3.1413-2	9.4229-3	2.4925-2	7.4255-3	2.4781-2	4.2046-3	2.7191-2	9.8545-4	2.9624-2	2.5973-4	2.8530-2	5.8720-3	2.0159-2
13	6.0710-2	1.8172-2	1.8992-2	2.0605-2	1.1159-2	2.0029-2	9.6300-3	1.8092-2	1.1135-2	1.4768-2	1.5191-2	9.7822-3	2.1516-2	3.7907-3	2.8305-2
14	5.1488-2	2.7067-2	8.4944-3	2.8700-2	1.9247-3	2.7319-2	5.6975-4	2.6054-2	4.3335-4	2.5269-2	9.6076-4	2.4365-2	2.8265-3	2.2085-2	7.4375-3
15	4.2800-2	3.4179-2	2.0378-3	3.1063-2	2.8713-4	2.5598-2	2.2123-3	2.1912-2	3.6225-3	2.0405-2	3.8253-3	2.0855-2	2.8517-3	2.2876-2	1.1484-3
16	3.4994-2	3.8532-2	8.7146-8	2.8014-2	4.8324-3	1.7543-2	1.0459-2	1.1142-2	1.3917-2	7.9106-3	1.5617-2	6.8300-3	1.6040-2	7.4514-3	1.5229-2

$v' \backslash v''$	15	16	17	18	19	20	21	22	23	24	25	26	27
0	9.4399-5	1.6998-5	2.5662-6	3.2317-7	3.3706-8	2.8836-9	2.0060-10	1.1235-11	4.9590-13	1.9734-14	1.2528-16	2.0799-16	6.0389-16
1	3.4093-3	8.6291-4	1.8051-4	3.1206-5	4.4467-6	5.1967-7	4.9422-8	3.7943-9	2.3417-10	1.1095-11	3.4235-13	1.7361-14	1.8362-15
2	3.5672-2	1.3418-2	4.0462-3	9.8770-4	1.9604-4	3.1660-5	4.1495-6	4.3901-7	3.7194-8	2.4953-9	1.2993-10	5.0766-12	1.6469-13
3	1.1566-1	7.5428-2	3.5429-2	1.2869-2	3.6898-3	8.4423-4	1.5491-4	2.2805-5	2.6846-6	2.5109-7	1.8463-8	1.0484-9	4.4781-11
4	8.2940-2	1.2990-1	1.1518-1	6.9607-2	3.0949-2	1.0519-2	2.7898-3	5.8366-4	9.6722-5	1.2687-5	1.3110-6	1.0569-7	6.5395-9
5	5.9053-3	1.9816-2	9.2101-2	1.3044-1	1.0355-1	5.9379-2	2.4233-2	7.5072-3	1.7991-3	3.3659-4	4.9287-5	5.6330-6	4.9852-7
6	6.5313-2	4.8253-2	1.1414-3	3.3656-2	1.0676-1	1.2893-1	9.3406-2	4.6616-2	1.7039-2	4.7060-3	9.9785-4	1.6352-4	2.0707-5
7	6.1426-3	2.2266-2	6.8871-2	3.2748-2	8.2227-4	5.5913-2	1.2159-1	1.2168-1	7.6253-2	3.3343-2	1.0698-2	2.5833-3	4.7525-4
8	3.6103-2	4.2274-2	2.6210-4	5.9363-2	6.5671-2	1.4529-2	1.1143-2	8.4197-2	1.3048-1	1.0693-1	5.7008-2	2.1469-2	5.9411-3
9	2.8104-2	2.2605-3	4.7481-2	2.6029-2	3.7874-3	5.7749-2	5.1100-2	1.5156-3	3.5628-2	1.1196-1	1.2836-1	8.5750-2	3.8429-2
10	1.9442-3	4.1368-2	1.2208-2	1.4639-2	5.0441-2	8.0584-3	2.0857-2	6.7486-2	2.7168-2	3.6116-3	7.1407-2	1.3023-1	1.1361-1
11	3.3332-2	1.2354-2	1.3818-2	3.9436-2	8.0111-4	3.4402-2	3.8620-2	3.0367-5	4.6607-2	5.9145-2	5.2553-3	2.7301-2	1.0818-1
12	1.9980-2	5.9996-3	3.5262-2	8.7432-4	3.1374-2	2.3984-2	4.6994-3	4.8807-2	1.6185-2	1.2230-2	6.5677-2	3.3467-2	1.8068-3
13	2.4929-5	3.0604-2	4.9631-3	2.1798-2	2.2934-2	4.3865-3	4.0537-2	5.0635-3	2.5210-2	4.3736-2	5.0077-4	4.1055-2	6.1736-2
14	1.6863-2	1.6093-2	8.2915-3	2.7504-2	5.7058-4	3.3945-2	5.1767-3	2.3053-2	3.0040-2	1.4968-3	4.6125-2	2.0152-2	9.6071-3
15	2.5634-2	1.2963-6	2.7191-2	2.1633-3	2.4151-2	1.1404-2	1.3699-2	2.7986-2	1.2270-3	3.8910-2	8.1905-3	2.1206-2	4.5013-2
16	9.9010-3	1.2818-2	1.4342-2	8.4316-3	2.1752-2	2.8302-3	2.9246-2	6.9650-5	3.0929-2	8.5040-3	1.8791-2	3.2031-2	8.2402-4

parabola. The  $N_2^+$  Meinel and  $N_2$  Second Positive systems have nearly equal  $\Delta r_e$  values (0.061Å, 0.064Å, respectively) and exhibit somewhat wider primary parabolas and the start of a secondary parabola. The  $N_2$  First Positive system exhibits one primary and two subsidiary parabolas all of which are relatively narrow as  $\Delta r_e=0.081$ Å for this system. Four wider subsidiary parabolas and one wider primary parabola whose vertex just avoids the (0,0) band are possessed by the  $N_2$  Lyman-Birge-Hopfield system ( $\Delta r_e=0.13$ Å). The extreme pattern of behavior (for  $\Delta r_e=0.196$ Å) is shown by the  $N_2$  Vegard-Kaplan system which possesses six very

wide subsidiary parabolas and one wide primary parabola.

A steady diminution in Franck-Condon factors with increasing  $v'$  is seen in the tables for the  $N_2$   $X^1\Sigma$ ,  $N_2^+$   $X^2\Sigma$ ,  $N_2^+$   $A^2\Pi$ ,  $N_2^+$   $B^2\Sigma$  ionization excitations.

The author's sincere thanks are due to Miss I. Stegun and Miss R. Zucker of the NBS Computation Laboratory for writing and testing the program, and to Mr. W. R. Jarman for providing details of the method of computation in a form suitable for programming.

TABLE 5. *Franck-Condon factors to high vibrational quantum numbers for bands of the N<sub>2</sub> Lyman-Birge-Hopfield (a<sup>1</sup>Π<sub>g</sub> – X<sup>1</sup>Σ<sub>g</sub><sup>+</sup>) system*

$v' \backslash v''$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	4.3147-2	1.5170-1	2.4766-1	2.4924-1	1.7313-1	8.8079-2	3.3993-2	1.0171-2	2.3917-3	4.4536-4	6.5886-5	7.7400-6	7.1926-7	5.2477-8
1	1.1623-1	1.9315-1	8.0487-2	4.0177-4	8.7320-2	1.8508-1	1.7516-1	1.0324-1	4.2495-2	1.2894-2	2.9684-3	5.2658-4	7.2502-5	7.7577-6
2	1.7128-1	9.6767-2	3.2760-3	1.0744-1	8.5983-2	8.5541-4	6.4509-2	1.6401-1	1.6142-1	9.4062-2	3.7178-2	1.0603-2	2.2534-3	3.6286-4
3	1.8348-1	1.2119-2	7.5543-2	6.9313-2	3.6061-3	9.5107-2	6.5801-2	1.4814-4	7.8397-2	1.6324-1	1.4306-1	7.5175-2	2.6753-2	6.8242-3
4	1.6026-1	6.3914-3	9.6009-2	5.8118-4	7.7442-2	3.7353-2	1.6885-2	9.6672-2	3.6152-2	9.1675-3	1.0847-1	1.6260-1	1.1787-1	5.3039-2
5	1.2140-1	4.7060-2	4.6680-2	3.3922-2	5.6701-2	8.3644-3	7.8822-2	7.9121-3	4.6748-2	8.5363-2	7.9576-3	3.9113-2	1.4016-1	1.4985-1
6	8.2871-2	8.5419-2	4.5383-3	7.2885-2	2.7951-3	6.3473-2	1.4846-2	4.0420-2	5.5021-2	2.0104-3	7.8737-2	5.1442-2	1.6276-3	8.7172-2
7	5.2304-2	9.9714-2	5.7959-3	5.6582-2	1.7174-2	4.6938-2	2.2779-2	5.7309-2	1.0399-3	6.7550-2	1.5376-2	3.2095-2	8.3544-2	1.1954-2
8	3.1090-2	9.2157-2	3.3721-2	1.8257-2	5.3397-2	4.2086-3	5.4396-2	4.7475-3	5.0309-2	1.7512-2	3.1436-2	5.5586-2	1.0426-3	7.3166-2
9	1.7644-2	7.3457-2	6.1052-2	1.3142-4	5.4846-2	9.2341-3	3.9713-2	1.5872-2	3.8686-2	1.1738-2	5.0739-2	1.2464-3	6.3084-2	1.2393-2
10	9.6608-3	5.2852-2	7.3781-2	9.7471-3	2.7952-2	3.9179-2	5.0804-3	4.7431-2	9.4200-4	4.9335-2	1.8434-3	4.9046-2	1.1514-2	3.5553-2
11	5.1456-3	3.5298-2	7.1626-2	3.2004-2	4.5490-3	4.9179-2	4.9041-3	3.4460-2	1.7415-2	2.5184-2	2.2571-2	2.6982-2	1.8394-2	4.0950-2
12	2.6837-3	2.2297-2	6.0437-2	5.1009-2	1.1577-3	3.2993-2	2.8601-2	5.8283-3	4.1688-2	2.1007-5	4.3320-2	5.4143-4	4.5667-2	6.3378-5
13	1.3780-3	1.3502-2	4.6253-2	5.9538-2	1.3977-2	1.0905-2	4.2374-2	2.3212-3	3.0711-2	1.7579-2	1.6377-2	2.8647-2	1.0683-2	3.3026-2
14	6.9962-4	7.9172-3	3.2962-2	5.7972-2	3.1418-2	2.5375-4	3.4721-2	2.0400-2	6.6949-3	3.6796-2	1.3573-4	3.6255-2	4.5241-3	3.3505-2
15	3.5254-4	4.5299-3	2.2268-2	5.0057-2	4.4474-2	4.2788-3	1.6705-2	3.5353-2	7.8834-4	2.8085-2	1.6663-2	1.1055-2	3.0416-2	2.8417-3
16	1.7686-4	2.5446-3	1.4443-2	3.9684-2	4.9779-2	1.6781-2	3.0392-3	3.4144-2	1.3841-2	7.8066-3	3.2518-2	3.9481-4	3.0054-2	8.7665-3
$v' \backslash v''$	14	15	16	17	18	19	20	21	22	23	24	25	26	27
0	2.9709-9	1.2832-10	4.1106-12	9.9345-14	3.3615-15	3.0101-17	4.3574-16	4.8245-16	1.0952-16	1.1648-15	2.7283-16	6.6761-16	1.8359-15	2.0131-16
1	6.4253-7	4.0808-8	1.9606-9	7.0520-11	1.8302-12	2.3676-14	4.1986-16	8.6253-16	4.5937-16	1.5854-16	9.8175-16	2.1260-16	2.0687-16	4.9335-16
2	4.4589-5	4.1814-6	2.9740-7	1.5857-8	6.2259-10	1.7511-11	3.5988-13	1.8946-15	8.4255-17	6.9250-16	9.1179-17	1.7731-18	1.1287-17	2.1375-16
3	1.2847-3	1.8106-4	1.9191-5	1.5250-6	8.9997-8	3.8880-9	1.1838-10	2.3433-12	3.3098-14	3.4689-16	1.1776-15	2.3464-16	1.0360-15	1.2129-15
4	1.6319-2	3.5966-3	5.8124-4	6.9568-5	6.1694-6	4.0236-7	1.9017-8	6.3003-10	1.3674-11	2.6901-13	5.5744-15	7.6479-16	4.3130-16	2.2953-16
5	8.7152-2	3.2616-2	8.4426-3	1.5660-3	2.1160-4	2.0924-5	1.5064-6	7.7871-8	2.8156-9	6.8393-11	1.2525-12	8.1166-15	1.5217-15	1.7040-15
6	1.5686-1	1.2213-1	5.6318-2	1.7212-2	3.6717-3	5.6045-4	6.1776-5	4.9073-6	2.7756-7	1.0921-8	2.8774-10	5.2210-12	5.7443-14	3.3133-15
7	3.1952-2	1.3391-1	1.4800-1	8.5723-2	3.1173-2	7.6735-3	1.3248-3	1.6284-4	1.4270-5	8.8267-7	3.7741-8	1.0816-9	2.0109-11	1.5562-13
8	4.9484-2	1.9439-3	8.9062-2	1.5546-1	1.1640-1	5.0917-2	1.4532-2	2.8438-3	3.9011-4	3.7718-5	2.5518-6	1.1851-7	3.6551-9	6.9987-11
9	3.4393-2	7.8477-2	7.5768-3	4.0354-2	1.4013-1	1.4178-1	7.5755-2	2.5231-2	5.6135-3	8.6061-4	9.1860-5	6.7969-6	3.4248-7	1.1363-8
10	4.4981-2	3.7797-3	7.6180-2	3.7765-2	7.2487-3	1.0525-1	1.5492-1	1.0330-1	4.0497-2	1.0285-2	1.7656-3	2.0824-4	1.6856-5	9.2168-7
11	5.2590-3	6.1266-2	4.7036-3	4.6677-2	6.8721-2	1.1699-3	6.1404-2	1.5085-1	1.2947-1	6.0438-2	1.7612-2	3.3944-3	4.4294-4	3.9248-5
12	5.0482-2	3.2684-3	4.6130-2	3.0377-2	1.3531-2	7.9458-2	2.0142-2	2.2846-2	1.2880-1	1.4915-1	8.4181-2	2.8335-2	6.1512-3	8.8962-4
13	1.2956-2	3.0547-2	2.6337-2	1.6466-2	5.4613-2	8.1207-7	6.4021-2	5.0336-2	1.9290-3	9.3245-2	1.5734-1	1.0964-1	4.2991-2	1.0555-2
14	7.7462-3	3.6321-2	5.5526-3	4.6588-2	2.0058-4	5.6134-2	1.2476-2	3.3442-2	7.3625-2	3.9385-3	5.3209-2	1.5075-1	1.3357-1	6.1671-2
15	3.6895-2	9.0014-4	4.0821-2	1.7892-3	4.3017-2	9.5126-3	3.4975-2	3.8010-2	7.2387-3	7.7082-2	2.4771-2	1.9622-2	1.2914-1	1.5192-1
16	2.1658-2	1.7875-2	1.7293-2	2.2493-2	1.9805-2	2.0646-2	3.2328-2	9.6230-3	5.5729-2	5.1842-4	5.9387-2	5.2688-2	1.6734-3	9.6082-2

TABLE 6.—*Franck-Condon factors to high vibrational quantum numbers for the  $N_2^+$  Meinel ( $A^2\Pi-X^2\Sigma$ ) band system*

$v' \backslash v''$	0	1	2	3	4	5	6	7	8	9	10
0	4.7507-1 ●	3.7976-1 ●	1.2261-1	2.0552-2	1.9140-3	9.8223-5	2.5730-6	2.8280-8	7.1881-11	1.3034-14	1.3712-16
1	3.2551-1	3.1147-2	3.3577-1 ●	2.3675-1	6.2361-2	7.9463-3	5.1634-4	1.6252-5	2.0531-7	5.6007-10	1.7873-13
2	1.3597-1	2.2454-1	2.1369-2	1.8509-1 ●	2.9463-1	1.1706-1	1.9723-2	1.5814-3	5.8634-5	8.3781-7	2.4027-9
3	4.5258-2	1.9900-1	7.9737-2	1.0485-1	6.2736-2	2.9286-1	1.7382-1	3.7932-2	3.6854-3	1.5859-4	2.5305-6
4	1.3290-2	1.0318-1	1.7446-1	7.1951-3	1.5534-1	5.8075-3	2.4791-1	2.2299-1	6.2282-2	7.2377-3	3.5730-4
5	3.6240-3	4.1446-2	1.3947-1	1.0776-1	6.9178-3	1.5087-1	5.6092-3	1.8156-1	2.5782-1	9.1651-2	1.2619-2
$v' \backslash v''$	11	12	13	14	15	16	17	18	19	20	21
0	2.7562-18	1.4459-16	1.0180-16	9.5982-17	6.8800-16	9.3860-17	4.4501-16	1.0922-15	1.5305-16	4.8250-16	1.4032-15
1	4.6499-15	2.7039-16	7.7745-16	5.7731-16	5.1409-17	4.8211-16	1.5310-16	2.8998-17	1.4699-16	1.5527-17	3.4885-17
2	1.2992-12	3.4067-15	9.4675-16	6.8119-18	9.2846-17	2.6601-18	5.3240-17	6.4896-18	7.2136-19	3.6857-19	3.3878-17
3	7.5480-9	6.1950-12	6.6985-15	5.2723-15	3.0122-16	8.5199-16	2.3340-16	4.6723-16	3.4614-16	1.6626-16	6.1908-16
4	6.3015-6	1.9287-8	2.2829-11	6.3533-15	4.6511-15	3.5759-15	1.4709-16	1.5949-15	7.9021-16	1.9054-16	6.2021-16
5	7.0813-4	1.3686-5	4.2478-8	7.2533-11	1.2025-14	1.1015-14	1.3299-15	1.9820-16	3.6896-16	1.1160-15	3.3943-16

TABLE 7. *Franck-Condon factor array to large vibrational quantum numbers for the  $N_2^+$  first negative ( $B^2\Sigma-X^2\Sigma$ ) band system*

$v' \backslash v''$	0	1	2	3	4	5	6	7	8	9	10
0	6.5094-1 ●	2.5883-1	7.0162-2	1.5997-2	3.2972-3	6.3420-4	1.1549-4	1.9998-5	3.2805-6	5.0335-7	7.0274-8
1	3.0144-1	2.2260-1	2.8598-1 ●	1.3242-1	4.2726-2	1.1403-2	2.6998-3	5.8613-4	1.1849-4	2.2393-5	3.9313-6
2	4.5371-2	4.0599-1	5.0646-2	2.2901-1	1.6535-1	7.1133-2	2.3623-2	6.6908-3	1.6951-3	3.9336-4	8.4449-5
3	2.2475-3	1.0562-1	4.1572-1	2.1005-3	1.5566-1	1.7060-1	9.4514-2	3.8008-2	1.2612-2	3.6675-3	9.6386-4
4	1.4521-5	6.9353-3	1.6604-1	3.7922-1	6.7256-3	9.2901-2	1.5692-1	1.0964-1	5.2363-2	2.0033-2	6.5985-3
5	4.6340-7	3.9858-5	1.3395-2	2.2051-1	3.3100-1	2.9248-2	4.8153-2	1.5327-1	1.1606-1	6.4883-2	2.8257-2
6	9.4819-9	3.0877-6	5.7286-5	2.0691-2	2.6731-1	2.8304-1	5.3307-2	2.0444-2	1.0647-1	1.1499-1	7.4424-2
7	6.4377-10	3.4169-8	1.1319-5	4.9298-5	2.7894-2	3.0677-1	2.4145-1	7.2364-2	5.9387-3	8.0835-2	1.0837-1
8	4.1998-13	5.7552-9	4.9506-8	3.0055-5	1.8070-5	3.4188-2	3.4009-1	2.0830-1	8.4684-2	4.7434-4	5.8649-2
9	1.2746-12	3.8766-12	2.6591-8	1.7424-8	6.4203-5	1.0666-6	3.8904-2	3.6856-1	1.8373-1	9.0654-2	5.5515-4
10	3.9487-14	1.0275-11	1.7096-10	8.3563-8	3.2950-8	1.1610-4	8.0941-5	4.1527-2	3.9326-1	1.6717-1	9.1407-2
11	2.8767-16	6.9997-13	3.8767-11	1.6762-9	1.9686-7	6.7150-7	1.8230-4	3.8725-4	4.1704-2	4.1493-1	1.5792-1
12	5.2594-16	2.4906-15	5.5598-12	7.8486-11	9.1842-9	3.5933-7	3.5224-6	2.5089-4	1.0871-3	3.9276-2	4.8379-1
13	7.4212-20	9.9105-16	1.0091-13	2.7547-11	5.7016-11	3.5189-8	4.9932-7	1.1720-5	3.0102-4	2.3614-3	3.4320-2
14	1.7188-16	9.8952-17	1.3821-15	9.9713-13	9.4783-11	1.3391-11	1.0410-7	4.7645-7	3.0048-5	3.0712-4	4.3643-3
15	1.8407-16	4.0381-17	2.6642-15	3.0960-15	7.6533-12	2.2485-10	1.1384-9	2.4797-7	2.0416-7	6.3998-5	2.4992-4
16	1.8159-17	5.4575-18	2.8290-15	6.5616-15	2.0529-14	3.6688-11	3.6122-10	8.9261-9	4.8200-7	1.5601-8	1.1714-4
17	9.5257-17	1.3459-16	1.3306-15	1.5689-15	5.3219-14	1.1348-12	1.2484-10	2.8360-10	3.8957-8	7.5533-7	1.3451-6
18	3.2282-16	6.4620-16	2.7374-17	3.0826-15	7.6808-15	4.8177-14	8.8436-12	3.2815-10	2.0518-15	1.2246-7	9.0082-7
19	3.0466-16	6.7846-16	1.1417-16	2.9053-15	3.1337-17	7.9994-14	3.1504-14	4.3885-11	6.1538-10	1.8661-9	3.0166-7
20	6.8290-17	1.0853-16	4.7800-17	1.3187-16	1.9387-16	1.0090-14	1.0265-13	1.2416-12	1.6333-10	7.0155-10	1.6088-8
21	2.0275-17	6.9310-17	2.1950-17	1.2681-15	2.4681-15	7.4429-16	7.4978-14	6.1713-14	1.2370-11	4.3238-10	2.3013-10
22	1.6131-16	5.1946-16	2.1950-17	2.0731-15	1.1194-15	1.1293-15	1.8187-14	1.0151-13	8.6085-15	6.8228-11	8.3364-10
23	3.0806-16	7.5001-16	1.8879-18	5.7561-16	2.8680-16	5.7740-16	8.7811-18	5.5815-14	2.4374-13	3.2389-12	2.4321-10
24	2.6276-16	4.0819-16	2.2567-16	3.9537-17	2.8615-15	3.7123-15	2.7915-15	2.7967-14	8.7905-14	7.2159-14	3.0774-11
25	7.3746-17	7.2516-17	5.2749-16	3.2663-16	2.1827-15	1.4253-15	3.9308-16	4.4983-15	1.0689-14	2.0665-13	1.5994-12
26	2.6573-18	1.4601-17	4.5026-16	2.0016-16	5.8352-17	3.0164-16	2.0010-15	1.9009-15	5.6924-16	7.6241-15	6.6945-17
27	1.0392-16	1.4208-16	8.1945-17	6.7362-18	8.7095-16	2.7341-15	4.5872-15	1.1201-14	2.1936-15	6.4900-14	3.8017-14
28	2.6226-16	2.7250-16	3.3132-17	2.8661-17	1.9786-15	2.3088-15	8.0611-16	1.1309-14	2.8800-14	1.4500-13	4.4470-14
29	2.6659-16	3.2371-16	3.0606-16	2.8421-17	1.0866-15	1.7611-16	1.8939-15	6.6923-15	9.2761-14	9.0932-14	1.4744-13

TABLE 7. *Franck-Condon factor array to large vibrational quantum numbers for the  $N_2^+$  first negative ( $B^2\Sigma-X^2\Sigma$ ) band system—Continued*

$v' \backslash v''$	11	12	13	14	15	16	17	18	19	20	21
0	8.4194-9	7.4706-10	2.7246-11	1.5572-12	5.7438-12	4.0732-12	1.9329-12	7.4934-13	2.4709-13	6.8549-14	1.6424-14
1	6.2862-7	8.7554-8	9.5331-9	5.6580-10	2.7584-13	4.1016-11	3.8757-11	2.1156-11	9.2253-12	3.5745-12	1.2773-12
2	1.6754-5	3.0305-6	4.8313-7	6.2656-8	5.2243-9	5.9097-11	1.2927-10	1.8980-10	1.2444-10	6.0838-11	2.5444-11
3	2.3237-4	5.1559-5	1.0440-5	1.8810-6	2.8382-7	3.0499-8	1.1388-9	1.9308-10	5.9679-10	4.8728-10	2.7405-10
4	1.9422-3	5.2074-4	1.2807-4	2.8771-5	5.7951-6	9.9985-7	1.3165-7	8.8002-9	5.3633-11	1.3336-9	1.4349-9
5	1.0429-2	3.4058-3	1.0071-3	2.7242-4	6.7369-5	1.5032-5	2.9233-6	4.5566-7	4.4113-8	3.8150-10	2.0816-9
6	3.6515-2	1.4962-2	5.3781-3	1.7413-3	5.1455-4	1.3912-4	3.4114-5	7.3925-6	1.3305-6	1.6789-7	6.7916-9
7	8.0508-2	4.4113-2	1.9912-2	7.8225-3	2.7556-3	8.8413-4	2.5973-4	6.9513-5	1.6626-5	3.3928-6	5.2530-7
8	9.8214-2	8.3188-2	5.0528-2	2.4953-2	1.0647-2	4.0574-3	1.4059-3	4.4632-4	1.2957-4	3.3924-5	7.7377-6
9	4.0751-2	8.6284-2	8.2879-2	5.5439-2	2.9771-2	1.3721-2	5.6271-3	2.0953-3	7.1529-4	2.2401-4	6.3751-5
10	3.6040-3	2.7101-2	7.3930-2	8.0178-2	5.8721-2	3.4099-2	1.6889-2	7.4191-3	2.9555-3	1.0800-3	3.6310-4
11	8.8180-2	7.8956-3	1.7195-2	6.2084-2	7.5740-2	6.0410-2	3.7741-2	1.9994-2	9.3675-3	3.9758-3	1.5488-3
12	1.5540-1	8.2045-2	1.2382-2	1.0343-2	5.1312-2	7.0185-2	6.0656-2	4.0577-2	2.2892-2	1.1393-2	5.1321-3
13	4.4957-1	1.5933-1	7.3838-2	1.6506-2	5.8379-3	4.1892-2	6.4044-2	5.9684-2	4.2558-2	2.5466-2	1.3412-2
14	2.7223-2	4.6145-1	1.6967-1	6.4180-2	2.0049-2	3.0433-3	3.3901-2	5.7743-2	5.7746-2	4.3701-2	2.7626-2
15	7.1648-3	1.8762-2	4.6808-1	1.8661-1	5.3547-2	2.3010-2	1.4302-3	2.7284-2	5.1592-2	5.5097-2	4.4065-2
16	1.3548-4	1.0676-2	1.0178-2	4.6761-1	2.1044-1	4.2359-2	2.5526-2	5.8439-4	2.1907-2	4.5804-2	5.1972-2
17	1.8672-4	2.0326-5	1.4586-2	3.2073-3	4.5788-1	2.4131-1	3.1069-2	2.7824-2	1.9790-4	1.7599-2	4.0509-2
18	7.5652-6	2.5870-4	3.5495-5	1.8313-2	1.9948-5	4.3662-1	2.7903-1	2.0265-2	3.0199-2	5.4549-5	1.4183-2
19	6.8357-7	2.4387-5	3.0557-4	3.9546-4	2.1029-2	3.0089-3	4.0194-1	3.2267-1	1.0750-2	3.2989-2	1.6955-5
20	5.9409-7	1.3334-7	5.8697-5	2.9260-4	1.3714-3	2.1794-2	1.4370-2	3.5292-1	3.7021-1	3.5923-3	3.6642-2
21	6.9591-8	9.1908-7	3.4595-7	1.1445-4	1.9932-4	3.2078-3	1.9842-2	3.5454-2	2.9044-1	4.1810-1	1.2231-4
22	5.0037-10	2.0893-7	1.0338-6	4.6938-6	1.8534-4	5.9069-5	5.9731-3	1.5008-2	6.5937-2	2.1796-1	4.6109-1
23	1.0306-9	1.0520-8	4.7806-7	6.4729-7	1.9630-5	2.4734-4	8.0611-6	9.3682-3	8.2444-3	1.0298-1	1.4206-1
24	6.5049-10	3.7077-10	5.8628-8	8.4343-7	2.1043-8	5.3360-5	2.5985-4	3.1278-4	1.2586-2	2.0152-3	1.4068-1
25	1.4345-10	1.1228-9	7.4423-10	2.0164-7	1.0704-6	1.3360-6	1.1017-4	1.8749-4	1.3264-3	1.4378-2	2.1219-4
26	1.4755-11	4.0176-10	8.5840-10	1.6039-8	4.8573-7	7.4719-7	1.0525-5	1.7906-4	5.2861-5	3.3301-3	1.3483-2
27	4.5578-14	4.7793-11	6.8930-10	8.1140-12	8.0276-8	8.1718-7	3.5258-8	3.6910-5	2.2192-4	1.7120-5	6.2372-2
28	2.0445-12	6.7752-13	1.4968-10	1.0541-9	2.5762-9	2.3357-7	8.9840-7	1.3616-6	8.5564-5	1.8547-4	4.3123-4
29	6.2502-12	1.9120-11	7.1972-12	9.0426-10	1.6712-9	2.2423-8	5.1404-7	4.9309-7	1.0523-5	1.4399-4	6.6619-5

TABLE 8. *Franck-Condon Factors for excitation of levels of  $N_2^+$  ( $X^2\Sigma_g^+$ ) from  $N_2(X^1\Sigma_g^+, v=0)$*

$v' \backslash v''$	0	$v' \backslash v''$	0
0	9.0236-1	11	1.9235-11
1	9.0636-2	12	3.4295-12
2	6.5080-3	13	6.5247-13
3	4.5437-4	14	1.3180-13
4	3.5005-5	15	2.6924-14
5	3.1250-6	16	4.5920-15
6	3.2409-7	17	6.9115-16
7	3.8320-8	18	3.8920-16
8	5.0548-9	19	8.9534-16
9	7.3154-10	20	1.5161-15
10	1.1463-10	21	8.5939-16

TABLE 9. *Franck-Condon Factors for the excitation of levels of  $N_2^+$  ( $A^2\Pi$ ) from  $N_2(X^1\Sigma_g^+, v=0)$*

$v' \backslash v''$	0
0	2.4468-1
1	3.1069-1
2	2.2530-1
3	1.2358-1
4	5.7340-2
5	2.3877-2



TABLE 10. *Franck-Condon Factors for the excitation of levels of  $N_2^+(B^2\Sigma_u^+)$  from  $N_2(X^1\Sigma_g^+, v=0)$*

$v'' \backslash v'$	0	$v'' \backslash v'$	0
0	8.9119-1	15	1.7921-17
1	1.0703-1	16	2.8420-20
2	1.7514-3	17	2.9486-17
3	2.6855-5	18	1.0171-16
4	1.7966-6	19	1.3940-16
5	6.5482-8	20	2.0681-17
6	1.0485-9	21	3.5482-17
7	3.3727-10	22	1.7063-16
8	5.2112-12	23	2.6721-16
9	1.8534-13	24	1.3949-16
10	4.9245-14	25	5.5412-19
11	5.0633-17	26	9.7988-17
12	1.0188-15	27	3.1142-16
13	3.3279-16	28	4.3268-16
14	5.4866-17	29	2.8228-16

## 5. Bibliography

- Bates, D. R. (1949), *Proc. Roy. Soc. [A]* **196**, 217.  
 Bates, D. R. (1952), *Monthly Notices Roy. Astron. Soc.* **112**, 614.  
 Biberman, L. M., and Yakubov, I. T. (1960), *Opt. and Spect.* **8**, 155.  
 Condon, E. U. (1926), *Phys. Rev.* **32**, 858.  
 Dixon, R. N., and Nicholls, R. W. (1958), *Can. J. Phys.* **36**, 127.  
 Dunham, J. L. (1932), *Phys. Rev.* **41**, 713, 721.  
 Fallon, R. J., Tobias, R., and Vanderslice, J. T. (1961), *J. Chem. Phys.* **34**, 167.  
 Fraser, P. A. (1954), *Proc. Phys. Soc. [A]* **67**, 939.  
 Fraser, P. A., and Jarman, W. R. (1953), *Proc. Phys. Soc. [A]* **66**, 1145.  
 Fraser, P. A., Jarman, W. R., and Henderson, G. P. (1959), *Scientific Report No. 2*, Contract AF 19 (604)-4560, University of Western Ontario.  
 Fraser, P. A., Jarman, W. R., and Nicholls, R. W. (1954), *Astrophys. J.* **119**, 286.  
 Gaydon, A. G., and Pearce, R. W. B. (1939), *Proc. Roy. Soc. [A]* **173**, 37.  
 Hebert, G. R., and Nicholls, R. W. (1961), *Can. J. Phys.* (in press).  
 Herzberg, G. (1950), *The Spectra of Diatomic Molecules* (D. Van Nostrand Co., Inc., New York, N.Y.).  
 Hutchisson, E. H. (1930), *Phys. Rev.* **36**, 410.  
 Hutchisson, E. H. (1931), *Phys. Rev.* **37**, 45.  
 Jarman, W. R. (1959), *J. Chem. Phys.* **31**, 1137.  
 Jarman, W. R. (1960), *Can. J. Phys.* **38**, 217.  
 Jarman, W. R., and Fraser, P. A. (1953), *Proc. Phys. Soc. [A]* **66**, 1153.  
 Jarman, W. R., and Nicholls, R. W. (1954), *Can. J. Phys.* **32**, 201.  
 Jarman, W. R., and Ebisuzaki, R., and Nicholls, R. W. (1960), *Can. J. Phys.* **38**, 510.  
 Jarman, W. R., Fraser, P. A., and Nicholls, R. W. (1953), *Astrophys. J.* **118**, 228.  
 Jarman, W. R., Fraser, P. A., and Nicholls, R. W. (1955), *Astrophys. J.* **122**, 55.  
 Losev, S. A. (1958), *Nauk Doklady Vyschei Shkoly, Fis Mat Nauchi* **5**.  
 Montgomery, C. E., and Nicholls, R. W. (1951), *Phys. Rev.* **82**, 565.  
 Mulliken, R. S. (1959), *The Threshold of Space*, p. 169 (Ed. M. Zelickoff, Pergamon Press, Inc., New York, N.Y.).  
 Morse, P. M. (1929), *Phys. Rev.* **34**, 57.  
 Nicholls, R. W. (1950), *Phys. Rev.* **77**, 421.  
 Nicholls, R. W. (1956), *Proc. Phys. Soc.* **69**, 741.  
 Nicholls, R. W. (1958), *Ann. géophys.* **14**, 208.  
 Nicholls, R. W. (1961), unpublished work.  
 Nicholls, R. W., and Jarman, W. R. (1956), *Proc. Phys. Soc.* **69**, 253.  
 Nicholls, R. W., and Jarman, W. R. (1959), *Proc. Phys. Soc.* **74**, 133.  
 Nicholls, R. W., Fraser, P. A., and Jarman, W. R. (1959), *Combustion and Flame* **3**, 13.  
 Nicholls, R. W., Fraser, P. A., Jarman, W. R., and McEachran, R. P. (1960), *Astrophys. J.* **131**, 399.  
 Ortenberg, F. S. (1960), *Opt. and Spect.* **9**, 82.  
 Pillow, M. E. (1949), *Proc. Phys. Soc. [A]* **62**, 237.  
 Pillow, M. E. (1950), *Proc. Phys. Soc. [A]* **63**, 940.  
 Pillow, M. E. (1951), *Proc. Phys. Soc. [A]* **64**, 772.  
 Pillow, M. E. (1952), *Proc. Phys. Soc. [A]* **65**, 858.  
 Pillow, M. E. (1953a), *Proc. Phys. Soc. [A]* **66**, 1064.  
 Pillow, M. E. (1953b), *Mem. Roy. Soc. Liege* **13**, 145.  
 Pillow, M. E. (1954), *Proc. Phys. Soc. [A]* **67**, 847.  
 Pillow, M. E. (1955), *Proc. Phys. Soc. [A]* **68**, 547.  
 Pillow, M. E., and Rowlatt, A. L. (1960), *Proc. Phys. Soc.* **75**, 162.  
 Robinson, D., and Nicholls, R. W. (1958), *Proc. Phys. Soc.* **71**, 957.  
 Robinson, D., and Nicholls, R. W. (1960), *Proc. Phys. Soc.* **75**, 817.  
 Tobias, R., Fallon, R. J., and Vanderslice, J. T. (1960), *J. Chem. Phys.* **33**, 1638.  
 Turner, R. G., and Nicholls, R. W. (1951), *Phys. Rev.* **82**, 290.  
 Turner, R. G., and Nicholls, R. W. (1954a), *Can. J. Phys.* **32**, 468.  
 Turner, R. G., and Nicholls, R. W. (1954b), *Can. J. Phys.* **32**, 475.  
 Vanderslice, J. T., Mason, E. A., and Maisch, W. G. (1960a), *J. Chem. Phys.* **32**, 515.  
 Vanderslice, J. T., Mason, E. A., and Maisch, W. G. (1960b), *J. Chem. Phys.* **33**, 614.  
 Vanderslice, J. T., Mason, E. A., Maisch, W. G., and Lip-pincott, E. R. (1959), *J. Mol. Spectroscopy* **3**, 17.  
 Vanderslice, J. T., Mason, E. A., Maisch, W. G., and Lip-pincott, E. R. (1960), *J. Mol. Spectroscopy* **5**, 83.  
 Wallace, L. V., and Nicholls, R. W. (1955), *J. Atmospheric and Terrest. Phys.* **7**, 101.  
 Wu, T. Y. (1952), *Proc. Phys. Soc. [A]* **65**, 965.  
 Wyller, A. (1953), *Mem. Roy. Soc. Liege* **13**, 917.  
 Wyller, A. (1958), *Astrophys. J.* **127**, 763.  
 Yakubov, I. T. (1960), *Opt. and Spect.* **9**, 212.

(Paper 65A5-126)

# Publications of the National Bureau of Standards

(Including papers in outside journals)

## Selected Abstracts

**Theory of an accurate intermediary orbit for satellite astronomy.** J. P. Vinti, *J. Research NBS* **65B** (Math. and Math. Phys.) No. 3, 169 (July-September 1961) 70 cents.

This paper derives an accurate intermediary orbit of an artificial satellite of an oblate planet. The drag-free motion takes place under the action of a gravitational potential which fits the even zonal harmonics exactly through the second and approximately through the fourth, in the case of the earth. This potential leads to separability of the Hamilton-Jacobi equation.

Two alternative sets of orbital elements are set forth. The first set is related directly to initial conditions, but requires numerical factoring of a certain quartic to evaluate some of the integrals. The second set, on the other hand, permits exact factoring of both quartics that appear, but is not related directly to initial conditions, so that its members can best be obtained by a least-square fit of the solution over many orbital revolutions.

The final solution is given in terms of certain uniformising variables, whose periodic terms are correct through the second order in the oblateness parameter and whose secular terms are exact, for the intermediary orbit. These exact solutions for the secular terms are expressed by means of certain rapidly converging series, with complete avoidance of elliptic integrals of the third kind.

**Prediction of symptoms of cavitation.** R. B. Jacobs, *J. Research NBS* **65C** (Eng. and Instr.) No. 3, 147 (July-September 1961) 70 cents.

An analysis which indicates some of the basic problems in cavitation and which may permit the prediction of cavitation characteristics of hydraulic equipment is presented. Some experimental results are discussed and are compared with the results of the analysis.

It is concluded that the analysis may be applicable to the prediction of symptoms of cavitation (changes in performance characteristics due to the presence of cavitation), but that more information related to metastability, nucleation, and vapor-phase dynamics is required.

**On the nature of the crystal field approximation.** C. M. Herzfeld and H. Goldberg, *J. Chem. Phys.* **34**, No. 2, 643-651 (Feb. 1961).

A new method is developed for the treatment of molecular interactions, and is applied to a system consisting of a hydrogen atom in a  $2p$  state and a hydrogen molecule in the ground state. The interaction of these two species is calculated using ordinary crystal field theory and also the new method. A comparison of the results shows some of the shortcomings of the conventional crystal field theory, and provides corrections to it. The new method consists of (1) expanding all electron terms of the total Hamiltonian for the system which involve interactions between the atom and the molecule, thus transforming the interaction Hamiltonian into sums of products of one-electron operators, and (2) of using properly antisymmetrized wave functions made up of products of atom and molecule eigenfunctions. The calculations show the effect of the neglect of overlap and exchange in ordinary crystal field theory.

**The electromagnetic fields of a dipole in the presence of a thin plasma sheet.** J. R. Wait, *Appl. Sci. Research* **8**, Sec. B, 397-417 (1960).

The problem of electric and magnetic dipoles located near a thin planar slab or sheet of ionized material is considered. A constant and uniform magnetic field is impressed on the slab. Under the assumption that the thickness of the slab is very small, expressions for the resultant fields are obtained. As a result of the anisotropy of the sheet it is indicated that the fields are elliptically polarized in general. On carrying out a

saddle-point evaluation of the integrals in the formal solution it is shown that the far fields may be split into "radiation" and "surface wave" components. The dependence of the radiation pattern and the surface wave characteristics on electron density, collision frequency, and the impressed magnetic field is illustrated.

**Calculation of properties of magnetic deflection systems.** S. Penner, *Rev. Sci. Instr.* **32**, No. 2, 150-160 (Feb. 1961).

A convenient matrix method for calculating properties of magnetic deflection systems is presented. This method is applicable to particle beams of small spatial and angular extent, and small energy spread. Equations for quadrupole lenses and for deflecting magnets are given. Examples are given to show the procedure for calculating the parameters of magnet systems.

**Departures from the Saha equation for ionized helium. I. Condition of detailed balance in the resonance lines.** R. N. Thomas and J. B. Zirker, *Astrophys. J.* **133**, No. 2, 588-595 (Mar. 1961).

Conditions for the validity of the assumption of detailed balance in the Lyman lines of He II are investigated. An opacity of  $10^6$  in Lyman- $\alpha$  is required, which implies high opacity in the subordinate lines and resonance continuum. The  $b_{\nu}$ -factors are computed, including the transfer problem in the subordinate lines and resonance continuum.

**Arc source for high temperature gas studies.** J. B. Shumaker, Jr., *Rev. Sci. Instr.* **32**, No. 1, 65-67 (Jan. 1961).

A wall-stabilized water-cooled copper ring type arc source is described which operates stably in apparently any gas for periods of an hour or more at currents up to 100 Amperes. Arc temperatures obtained spectroscopically by absolute line intensity and line profile methods are given for the arc in nitrogen at 92 Amperes.

**Electron diffraction studies on solid  $\alpha$ -nitrogen.** E. M. Horl and L. Marton, *Acta Crystallographica* **14**, Pt. 1, 11-19 (Jan. 10, 1961).

A technique was developed for studying thin films of solidified permanent gases by means of electron transmission diffraction. It was applied to the investigation of thin films of solid  $\alpha$ -nitrogen at  $4^\circ$  and  $20^\circ$  K. The orientation of the microcrystallites on aluminum and Formvar substrates was investigated as a function of both the temperature of the substrate and the intensity of the molecular nitrogen beam during deposition. Further, the crystal structure and the cell dimensions of  $\alpha$ -nitrogen were reinvestigated. Faults in the stacking sequence of (111) planes, detected under certain conditions, were studied in some detail. Preferential crystal growth processes were observed to be induced by electron bombardment. The influence of conditions of deposition on these processes was also investigated.

**The absorption spectra of magnesium and manganese atoms in solid rare gas matrices.** O. Schnepp, *J. Phys. Chem. Solids* **17**, No. 3/4, 188-195 (1961).

The absorption spectra of magnesium and of manganese in solid rare gas matrices have been investigated at liquid helium temperature. An absorption system of magnesium near 2850 Å and two for manganese near 4000 Å and 2800 Å have been observed. These absorptions lie very close to the wavelengths of allowed atomic transitions and it is concluded on the basis of the evidence presented here that the absorbing species are atoms which are, in general, trapped at more than one type of site. The crystal field causes the removal of the orbital degeneracy of the excited atomic  $P$  states; the splitting energy is observed to be of the order of  $300\text{ cm}^{-1}$ . It is proposed that the removal of the degeneracy is due to asymmetric environments of the trapping site. Possible sites are discussed.

**Near infrared atmospheric transmission to solar radiation.** D. M. Gates, *J. Opt. Soc. Am.* **50**, No. 12, 1299-1304 (Dec. 1960).

Near infrared solar spectrum observations taken on October 15, 1954 with a double-pass NaCl prism spectrometer have been analyzed for transmission coefficients for the "selective" absorption factor and for the "continuum" factor. The analysis was carried out for 59 wavelength positions between 0.872 and 2.537  $\mu$ . The monochromatic data fit well the law  $^{\circ}nT = c_1(w)^{1/2}$  where  $w$  is the amount of water vapor in the optical path. The coefficient  $c_1$  is given as a continuous function of the wavelength. A coefficient of extinction for the "continuum" factor is also given.

**Observations on the chemiluminescent reaction of nitric oxide with atomic oxygen.** H. P. Broida, H. I. Schiff, and T. M. Sugden, *Trans. Faraday Soc.* **57**, No. 458, 259-265 (Feb. 1961).

A comparison of the absorption spectrum of NO<sub>2</sub> with the emission from the reaction of NO and O produced in various ways at pressures of a few mm of Hg, shows that the emitter is NO<sub>2</sub> in a metastable excited electronic state. Measurement of the short wavelength cut-off of the emission spectrum leads to a minimum value for  $\Delta H_0$  of NO<sub>2</sub> → NO + O of 71.2 ± 0.1 kcal/mole, in fair agreement with thermal data. The loss of discrete structure in the emission from the same reaction in high-temperature flames is ascribed to increasing complexity which could not be resolved.

The data are examined with respect to a system of potential-energy curves based on earlier work on the fluorescence and the spectroscopy of NO<sub>2</sub>, and it is concluded that the emission from the interaction of NO and O occurs as a result of a resonance transfer process. The available evidence on the kinetics of the reaction is discussed in the light of a mechanism based on the two excited states NO<sub>2</sub><sup>\*</sup> involved, and it is shown that the resonance transfer process between them may be rate-determining, both in the emission from NO + O, and in the fluorescence of NO<sub>2</sub>. The simple process NO + O → NO<sub>2</sub> +  $h\nu$  is largely discarded, both on spectroscopic and kinetic grounds.

**Vacuum ultraviolet photolysis of ethane: Molecular detachment of hydrogen.** H. Okabe and J. R. McNesby, *J. Chem. Phys.* **34**, No. 2, 668-669 (Feb. 1961).

The primary process of direct ethane photolysis by Xe radiation (1470 Å and 1295 Å) was studied at room temperature. The hydrogen and methane isotopic compositions from a mixture of C<sub>2</sub>H<sub>6</sub>-C<sub>2</sub>D<sub>6</sub> and from CH<sub>3</sub>CD<sub>3</sub> were measured mass-spectrometrically. The results show that, contrary to the previously proposed mechanism, almost all (>95%) of the hydrogen is formed intramolecularly, and preferentially from the same carbon atom. This may be compared with the mercury-sensitized photolysis of ethane where evidence indicates that almost all hydrogen is produced by an atomic process. Methane is also formed by a molecular process. Then the primary processes may be written as



No ethane-d<sub>6</sub> was found from CH<sub>3</sub>CD<sub>3</sub> photolysis indicating that no methyl radicals are formed in a primary process. The effect of ethylene on the composition of hydrogen isotopes from C<sub>2</sub>H<sub>6</sub>-C<sub>2</sub>D<sub>6</sub> mixtures was examined, since any atomic hydrogen produced may be reacting rapidly with ethylene. The sample was purified and very low conversion (0.01%) was made. The result shows almost no change in the composition of the hydrogen isotopes in the products, indicating that the hydrogen forming step is almost all molecular. It is suggested that higher hydrocarbons may be produced by the following reactions



**The effect of solvents of the  $\gamma$ -ray radiolysis of methyl acetate and acetone.** P. Ausloos, *J. Am. Chem. Soc.* **13**, No. 5, 1056-1060 (Mar. 1961).

The effect of cyclohexane and benzene on the radiolysis of CH<sub>3</sub>COOCH<sub>3</sub>, CH<sub>3</sub>COOCD<sub>3</sub>, and mixtures of CH<sub>3</sub>COCH<sub>3</sub>-

CD<sub>3</sub>COCD<sub>3</sub> has been investigated at 17°. In the absence of scavengers, cyclohexane markedly reduces the yield of CO whereas, in the presence of scavengers, it produces no effect. Benzene inhibits the decomposition of methyl acetate and acetone. Hydrogen is mostly formed in processes involving "hot" hydrogen atoms. The effect of the solvents on the yields of ethanes and some of the methanes indicates the importance of cage and intercage recombinations and disproportionations of the radicals in the  $\gamma$ -ray track. The radiolysis of a CH<sub>3</sub>COCH<sub>3</sub>-CD<sub>3</sub>COCD<sub>3</sub> mixtures at -195° is discussed briefly.

**Intramolecular rearrangements. I. *sec*-butyl acetate and *sec*-butyl formate.** R. Borkowski and P. Ausloos, *J. Am. Chem. Soc.* **83**, No. 5, 1053-1056 (Mar. 1961).

The photolysis of *sec*-butyl acetate and *sec*-butyl formate in the vapor, liquid, and solid phases has been studied at different temperatures and wavelengths. The distributions of the three butenes produced in the intramolecular-rearrangements were found to vary with temperature in all three phases. In the vapor phase, a change in wavelength has a pronounced effect on the ratios 1-butene/2-butenes and *cis*-2-butene/*trans*-2-butene. Both ratios increased with a decrease in wavelength. In the liquid phase photolysis, however, no dependence on wavelength was observed, but a drastic change of the butene distribution occurred in the short temperature range associated with the formation of the glassy state. A brief study of the pyrolysis of these compounds showed that, at temperatures greater than 300°, the temperature coefficient of the butene distribution was essentially zero.

**Hydrogen atom reactions with propene at 77°K. Disproportionation and recombination.** R. Klein and M. D. Scheer, *J. Phys. Chem.* **65**, 324-325 (1961).

The investigation of the reaction products of deuterium atoms with ordinary propene helps to establish the relative importance of the atom addition to the propyl radical with respect to disproportionation of two propyl radicals. The distribution of the isotope species among the propenes and propanes is calculated. It is concluded that the D atom addition to the propyl radical accounts for little, if any, of the propane formed, and the radical disproportionation reaction is the predominant one.

**The activation energy for hydrogen atom addition to propylene.** M. D. Scheer and R. Klein, *J. Phys. Chem.* **65**, 375-377 (1961).

The activation energy for H + CH<sub>3</sub>CH=CH<sub>2</sub> → CH<sub>3</sub>CHCH<sub>3</sub> has been measured in a study of the solid phase reaction. Hydrogen atoms, formed in the gas by dissociation on a hot tungsten ribbon, were allowed to diffuse through and react with thin dilute films of propylene in the 77 to 90°K temperature range. The activation energy was determined by measuring the rate of propylene depletion as a function of film temperature. A value of 1.5 kcal/mole was obtained. No significant isotope effect was observed when D instead of H atoms were used. A steric factor of 3 × 10<sup>-3</sup> is calculated for the gas phase reaction from the 300°K rate constant  $(2 \times 10^{11} \frac{\text{cc}}{\text{mole sec}})$  and an H, C<sub>3</sub>H<sub>6</sub> collision diameter of 5.5 Å.

**Interpretation of the appearance potentials of secondary ions.** M. B. Wallenstein and M. Krauss, *J. Chem. Phys.* **34**, No. 3, 929-936 (Mar. 1961).

The statistical theory of mass spectra assumes that the excitation energy in a molecule-ion is essentially equipartitioned prior to unimolecular decomposition. Some of the consequences of this assumption have been deduced and the results have been used to interpret the observed appearance potentials of secondary and tertiary ions of neo-pentane, *n*-butane, and *i*-butene. Although the results are not quantitative, it is evident that considerable vibrational energy is removed with the neutral fragment.

The problem of the detailed interpretation of appearance potentials would seem to be complicated by the possibility of considerable fluctuation of the vibrational energy distribution in the products of a decomposition from the most probable distribution, i.e., equipartition. Some general considerations

with regard to determination of minimum-energy decomposition paths and the significance of this study in relation to the statistical theory of mass spectra are also presented.

**Determination of crystallite size distributions from X-ray line broadening.** A. Bienenstock, *J. Appl. Phys.* **32**, No. 2, 187-189 (Feb. 1961).

The broadening of an 001 powder diffraction line due to a distribution of sizes of crystallites is discussed. A function of the intensity,  $P(h_3) \sin^2(\pi h_3)$ , is derived. Its cosine transform gives the size distribution directly. The first term of a series expansion of this distribution function corresponds to the expression previously obtained by Warren and Averbach. In addition this function gives less weight to the extremes of the diffraction line shape.

**The dependence of the melting temperature of bulk homopolymers on the crystallization temperature.** L. Mandelkern, *J. Polymer Sci.* **XLVII**, issue 149, 494-496 (July 1960).

Based on the hypothesis that during the crystallization from the melt of bulk homopolymers the length of a crystallite in the chain direction does not significantly exceed that of a critical size nucleus, a simple relation is derived between the melting temperature and the crystallization temperature. This relation is found to describe the experimental results of Wood and Bekkedahl on natural rubber, where a marked effect of the crystallization temperature on the melting temperature has been reported.

**The interfacial properties of polyesters at glass and water surfaces.** R. R. Stromberg, *Soc. Plastic Eng. J.* **15**, 882-886 (Oct. 1959).

Some interfacial factors related to adhesion were studied. Monomolecular layers of stearic acid and polyesters on liquid subphases are compared. Stearic acid yields a condensed type isotherm, indicating close packing and high cohesion. The monolayer of poly(ethylene adipate) at low surface pressure is the gaseous type, with very low cohesive forces, and at higher pressures, the liquid expanded type. Poly(ethylene succinate) remains in an expanded liquid state up to collapse pressure. Adsorption of polyesters from dilute solutions onto glass, silica, and alumina was also studied. Considerably more polymer was adsorbed from a poor solvent than from a good solvent. A negative temperature coefficient was observed for adsorption from the poor solvent and no temperature effect from the good solvent. This behavior was attributed, in part, to differences in the configuration of the polyester molecule in the two solvents.

### Other NBS Publications

**Journal of Research 65B (Math. and Math. Phys.) No. 3 (July-September 1961) 75 cents.**

Theory of an accurate intermediary orbit for satellite astronomy. J. P. Vinti. (See above abstract.)

Note on the "baffled piston" problem. F. Oberhettinger. Some results on non-negative matrices. M. Marcus, H. Mine, and B. N. Moys.

Probability inequalities of the Tchebycheff type. I. R. Savage.

**Journal of Research 65C (Engr. and Instr.) No. 3 (July-September 1961) 75 cents.**

Prediction of symptoms of cavitation. R. B. Jacobs. (See above abstract.)

Heating and cooling of air flowing through an underground tunnel. B. A. Peavy.

Stress-corrosion cracking of the AZ31B magnesium alloy. H. L. Logan.

Coatings formed on steel by cathodic protection and their evaluation by polarization measurements. W. J. Schwerdtfeger and R. J. Manuele.

Calibration of inductance standards in the Maxwell-Wien bridge circuit. T. L. Zapf.

Calibration of loop antennas at VLF. A. G. Jean, H. E. Taggart, and J. R. Wait.

Location of the plain of best average definition with low contrast resolution patterns. F. E. Washer and W. P. Tayman.

Influence of temperature and relative humidity on the photographic response to  $\text{Co}^{60}$  gamma radiation. M. Ehrlich.

**Journal of Research 65D (Radio Prop.) No. 5 (September-October 1961) 70 cents.**

Frequency dependence of D-region scattering at VHF. J. C. Blair, R. N. Davis, Jr., and R. C. Kirby.

Theoretical scattering coefficient for near vertical incidence from contour maps. H. S. Hayre and R. K. Moore.

Mutual interference between surface and satellite communication systems. W. J. Hartman and M. T. Decker.

VHF and UHF signal characteristics observed on a long knife-edge diffraction path. A. P. Barsis and R. S. Kirby.

Experimental study of inverted L-, T-, and related transmission-line antennas. S. Prasad and R. W. P. King.

Reflection from a sharply bounded ionosphere for VLF propagation perpendicular to the magnetic meridian. D. D. Crombie.

Resonance of the space between earth and ionosphere. H. Poeverlein.

Observed attenuation rate of ELF (region below 1 kc/s) radio waves. A. G. Jean, A. C. Murphy, J. R. Wait, and D. F. Wasmundt.

A note concerning the excitation of ELF electromagnetic waves. J. R. Wait.

Computation of whistler ray paths. I. Yabroff.

On the analysis of LF ionospheric radio propagation phenomena. J. R. Johler.

Ideal gas thermodynamic functions and isotope exchange functions for the diatomic hydrides, deuterides, and tritides, L. Haar, A. S. Friedman, and C. W. Beckett, NBS Mono. 20 (1961) \$2.75.

Bibliography of temperature measurement—January 1953 to June 1960, C. Halpern and R. J. Moffat, NBS Mono. 27 (1961) 15 cents.

Causes of variation in chemical analyses and physical tests of portland cement, B. L. Bean and J. R. Dize, NBS Mono. 28 (1961) 25 cents.

Thermal expansion of technical solids at low temperatures. A compilation from the literature, R. J. Corruccini and J. J. Gniewek, NBS Mono. 29 (1961) 20 cents.

Corrected optical pyrometer readings, D. E. Poland, J. W. Green, and J. L. Margrave, NBS Mono. 30 (1961) 55 cents.

Mean electron density variations of the quiet ionosphere—May 3, 1959, J. W. Wright, L. R. Wescott, and D. J. Brown, NBS TN40-3 (PB151399-3) (1960) \$1.50.

A survey of computer programs for chemical information searching, E. C. Marden and H. R. Koller, NBS TN85 (PB161586) (1961) \$2.25.

The Franck-Condon factor ( $q_{v''v'}$ ) array to high vibrational quantum numbers for the  $\text{O}_2(B^3\Sigma_u^- - X^3\Sigma_g^-)$  Schumann-Runge band system, R. W. Nicholls, *Can. J. Phys.* **38**, 1705-1711 (1960).

NBS—source of American Standards, W. A. Wildhack, *ISA J.* **8**, No. 2, 45-50 (Feb. 1961).

Ionospheric mapping by numerical methods, W. B. Jones and R. M. Gallet, *Telecommun. J.* **12**, 260-264 (Dec. 1960).

Titanium (III) chloride and titanium (III) bromide (titanium trichloride and titanium tribromide), J. M. Sherfey, Chapter IVB, Sec. 17, p. 57-61, Book, *Inorganic Syntheses*, by E. G. Rochow (McGraw-Hill Book Co., New York, N.Y., 1960).

Science news writing, D. M. Gates, J. M. Parker, *Science* **133**, No. 3447, 211-214 (Jan. 1961).

Integrated starlight over the sky, F. E. Roach and L. R. Megill, *Astrophys. J.* **133**, No. 1, 228-242 (Jan. 1961).

Elastomers for static seals at cryogenic temperatures, D. H. Weitzel, R. F. Robbins, G. R. Bopp, and W. R. Bjorklund, *Rev. Sci. Instr.* **31**, No. 12, 1350-1351 (Dec. 1960).

New standards for the space age, A. T. McPherson, *Astronautics* **6**, No. 1, 24-25; 50-54 (Jan. 1961).

Correlation of visual and subvisual auroras with changes in the outer Van Allen radiation zone, B. J. O'Brien, J. A. Van Allen, F. E. Roach, and C. W. Gartlein, *IGY Bull.*, No. 45, 1-16 (Mar. 1961).



- Spiral patterns in geophysics, V. Agy, *J. Atmospheric and Terrest. Phys.* **19**, 136-140 (1960).
- Iron (99.9+), G. A. Moore and T. R. Shives, *Metals Handb.* **1**, 1206-1212 (1961).
- Ionospheric absorption at times of auroral and magnetic pulsations, W. H. Campbell and H. Leinback, *J. Geophys. Research* **66**, 25-34 (Jan. 1961).
- Some properties of new or modified excitation sources, M. Margosbes, *Am. Soc. Testing Materials, Spec. Tech. Publ.* **259**, 46-58 (1959).
- Seasonal and day-to-day changes of the central position of the  $S_a$  overhead current system, S. Matsushita, *J. Geophys. Research* **65**, No. 11, 3835-3839 (Nov. 1960).
- Hydrogen sulfide precipitation of the elements from 0.2-0.5 normal hydrochloric acid, J. I. Hoffman, *Chemist-Analyst* **50**, No. 1, 30 and 32 (Mar. 1961).
- The mechanism of electrolytic deposition of titanium from fused salt media, W. E. Reid, *J. Electrochem. Soc.* **108**, No. 4, 393-394 (Apr. 1961).
- Studies of elevated temperature corrosion of type 310 stainless steel by vanadium compounds, H. L. Logan, *Corrosion* **17**, 109-111 (Apr. 1961).
- Deep penetration of radiation, U. Fano and M. J. Berger, *Proc. Symp. Appl. Math.* **XI**, 43-59 (1961).
- Deposition of iron from salts of fluoro-acids, J. H. Connor and V. A. Lamb, *Plating* **48**, No. 4, 388-389 (Apr. 1961).
- A simple low-temperature specimen holder for an X-ray diffractometer, D. K. Smith, *Norelco Reporter* **VII**, No. 1, 11-12 (Jan.-Feb. 1961).
- Stepless variable resistor for high currents, C. R. Yokley and J. B. Shumaker, Jr., *Rev. Sci. Instr.* **32**, No. 1, 6-8 (Jan. 1961).
- Microbalance techniques for high temperature application, R. F. Walker, *Book, Vacuum microbalance techniques*, edited by M. J. Katz, **I**, 87-110, (Plenum Press Inc., New York, N.Y., 1961).
- Investigation of bond in beam and pull-out specimens with high-yield-strength deformed bars, R. G. Mathey and D. Watstein, *J. Am. Concrete Inst.* **32**, No. 9, 1071-1090 (Mar. 1961).
- Shielding calculations for civil defense, C. Eisenhauer, *Health Phys.* **4**, No. 2, 129-132 (1960).
- A study of auroral coruscations, W. H. Campbell and M. H. Rees, *J. Geophys. Research* **66**, No. 1, 41-55 (Jan. 1961).
- The nature, cause and effect of the porosity in electrodeposits. VII. A microscopic examination of nickel-chromium coatings after atmospheric corrosion, D. W. Ernst and F. Ogburn, *Plating* **48**, No. 4, 491-497 (May 1961).
- Physical quantities proposed for radiation measurements, L. S. Taylor, *Proc. IXth Intern. Congress of Radiology*, July 23-30, 1959 (Munich, Germany), 1301-1302 (1960).
- Improving rectifier circuits, G. F. Montgomery, *Electronics* p. 86-87 (April 7, 1961).
- Fracture characteristics of notched tensile specimens of titanium and a titanium alloy, G. W. Geil and N. L. Carwile, *Materials Research and Standards (Am. Soc. Testing Materials, Philadelphia, Pa.)* **1**, No. 1, 16-21 (Jan. 1961).
- The robustness of life testing procedures derived from the exponential distribution, M. Zelen and M. C. Dannemiller, *Technometrics* **3**, No. 1, 29-49 (Feb. 1961).
- Absence of an isotope effect in the fractional recrystallization of alpha-D-glucose-1-t, H. S. Isbell, H. L. Frush, and N. B. Holt, *Anal. Chem.* **33**, 225-226 (Feb. 1961).
- Applications of graphs and Boolean matrices to computer programming, R. B. Marimont, *SIAM Rev.* **2**, No. 4, 259-268 (Oct. 1960).
- Comparison of national standards for roentgen measurement, H. O. Wyckoff, *IXth Intern. Congress of Radiology*, July 23-30, 1959 (Munich, Germany), 1315-1318 (1960).
- Environmental factors in a family-size underground shelter, R. P. Achenbach, *Proc. Meeting on Environmental Engineering in protective Shelters*, Feb. 8-10, 1960, *Natl. Acad. Sci.-Natl. Research Council*, Wash. D.C., 69-118 (1961).
- How to evaluate accuracy, W. J. Youden, *Materials Research and Standards (Am. Soc. Testing Materials, Philadelphia, Pa.)* **1**, No. 4, 268-271 (Apr. 1961).
- Improved standard for the calibration of vibration pickups, R. R. Bouche, *Exp. Mech.* 1-6 (Apr. 1961).
- Absorption and scattering of photons by holmium and erbium, E. G. Fuller and E. Hayward, *Proc. Intern. Conf. Nuclear Structure*, 760-766 (Kingston, Canada, 1960).
- Introductory remarks, U. Fano, *Conf. Coherence Properties of Electromagnetic Radiation, AFOSR-583*, Report on Conf., 31-38 (University of Rochester, The Institute of Optics, Rochester, N.Y., Apr. 1961).
- Available heat sinks for protected underground installations, B. A. Peavy, *Proc. Meeting on Environmental Engineering in Protective Shelters*, Feb. 8-10, 1960, *Natl. Acad. Sci.-Natl. Research Council*, Wash. D.C., 69-118 (1961).
- Studies of the low-temperature distillation of hydrogen isotopes, T. M. Flynn, *Cryogenics* **1**, No. 2, 1-5 (Dec. 1960); *Advances Cryogenic Engr.* **6**, 236-244 (1961).
- Inert enclosed pump for shaped flow of ultraclean solutions, M. W. Barnes and R. H. Noyce, *Rev. Sci. Instr.* **32**, No. 3, 353 (Mar. 1961).
- On some partial differential equations of Brownian motion of a free particle, A. Ghaffari, (Abstract) *Proc. Intern. Conf. on Partial Differential Equations and Continuum Mechanics* 348-350 (1961) (Univ. of Wisconsin, Madison, Wis., June 7-15, 1960).
- Radiation protection standards, L. S. Taylor, *Radiology* **74**, 824-831 (May 1960).
- Atomic beam frequency standards, R. C. Mockler, R. E. Beehler, and C. S. Snider, *IRE Trans. Instrumentation* **I-9**, No. 2, 120-132 (Sept. 1960).
- The development of more stable gage blocks, M. R. Meyerson, T. R. Young, and W. R. Ney, *ASTM Bull.* **1**, No. 5, 368-374 (May 1961).
- The power spectrum and its importance in precise frequency measurements, J. A. Barnes and R. C. Mockler, *IRE Trans. Instrumentation* **I-9**, No. 2, 149-155 (Sept. 1960).
- VLF phase perturbation associated with meteor shower ionization, C. J. Chilton, *J. Geophys. Research* **66**, No. 2, 379-383 (Feb. 1961).
- Tests for regression coefficients when errors are correlated, M. M. Siddiqui, *Annals Math. Stat.* **31**, No. 4, 929-938 (Dec. 1960).
- A comment on the NRL solar Lyman-Alpha results, J. T. Jefferies and R. N. Thomas, *Astrophys. J.* **133**, No. 2, 606-607 (Mar. 1961).
- The nation's electronic standards program: where do we now stand? H. W. Lance, *IRE Trans. Instrumentation* **I-9**, No. 2, 94-100 (Sept. 1960).
- Hard gallium alloys for use as low contact resistance electrodes and for bonding thermocouples into samples, G. C. Harmon, *Rev. Sci. Instr.* **31**, No. 7, 717-720 (July 1960).
- Scattering of photons by deformed nuclei, E. G. Fuller and E. Hayward, *Proc. Intern. Conf. Nuclear Structure*, 763-766 (Kingston, Canada, 1960).

*Publications for which a price is indicated (except for Technical Notes) are available only from the Superintendent of Documents, U.S. Government Printing Office, Washington 25, D.C. (foreign postage, one-fourth additional).*

*Technical Notes are available only from the Office of Technical Services, U.S. Department of Commerce, Washington 25, D.C. (Order by PB number). Reprints from outside journals and the NBS Journal of Research may often be obtained directly from the authors.*